# QoS Analysis of Cognitive Radio Channels with Perfect CSI at both Receiver and Transmitter

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***Abstract*—[1] In this paper, cognitive transmission under quality of service (QoS) constraints is studied. In the cognitive radio channel model, it is assumed that both the secondary receiver and the secondary transmitter know the channel fading coefficients perfectly and optimize the power adaptation policy under given constraints, depending on the channel activity of the primary users, which is determined by channel sensing performed by the secondary users. The transmission rates are equal to the instantaneous channel capacity values. A state transition model with four states is constructed to model this cognitive transmission channel. Statistical limitations on the buffer lengths are imposed to take into account the QoS constraints. The maximum throughput under these statistical QoS constraints is identified by finding the effective capacity of the cognitive radio channel. The impact upon the effective capacity of several system parameters, including the channel sensing duration, detection threshold, detection and false alarm probabilities, and QoS parameters, is investigated.**

## I. Introduction

With growing demand for spectrum use in wireless communications, efficient use of the spectrum is becoming necessary. On the other hand, recent studies show that licensed users are not utilizing the spectrum efficiently. This has led to much interest in cognitive radio systems. Since the basic principle in cognitive transmission is not to disturb the primary users of the spectrum, it is very important to detect the activities of the primary users and not cause any harmful interference to their communications. When the primary users are active, the secondary user should either avoid using the channel or spend low power not to exceed the noise power threshold of the primary users, whereas when the channel is free of the primary users, secondary users can use the channel without any constraints.

Considering a model which describes the busy and idle periods of a wireless LAN (WLAN), Geirhofer *et al.* in their studies [7] focused on dynamically sharing the spectrum in the time-domain by exploiting the whitespace between the bursty transmissions of a set of users, represented by an 802.11b based WLAN. They found that Pareto distribution provides an adequate fit for varying packet rates and they proposed a continuous-time semi-Markov model that captures the idle periods remaining between the bursty transmission of WLAN. The problem of maximally utilizing the spectrum opportunities in cognitive radio networks with multiple potential channels and the problem of optimal sensing order in multi-channel cognitive medium access control with opportunistic transmission are investigated in [8]. The authors in [9] studied the opportunistic secondary communication over a spectral pool of two independent channels and showed that the benefits of spectral pooling are lost in dynamic spectral environments.

In communication systems, it is very important to satisfy certain quality of service (QoS) constraints in order to guarantee an expected level of performance to the users. However, this is a very challenging task in wireless systems because of the random variations in channel conditions and the resulting random fluctuations in received power levels. Consequently, only statistical, rather than deterministic, service guarantees can most of the time be provided in wireless communications. Note that the situation is further exasperated in cognitive wireless systems due to additional random fluctuations in the transmitted power levels, depending on the presence or absence of primary users. For instance, when there are active primary users present, secondary users either cease the transmission or transmit only at lower power levels, while they can transmit at higher power levels in the absence of active primary users. Furthermore, cognitive radio can suffer from errors in channel sensing in the form of false alarms. Hence, it is of interest to analyze the performance of cognitive radio systems in the presence of QoS constraints.

The notion of effective capacity is a useful tool to obtain the maximum throughput levels in wireless systems under statistical QoS constraints. The application and analysis of effective capacity in various settings has attracted much interest. The effective capacity is defined by Wu and Negi in [4] as the maximum constant arrival rate that a given time-varying service process can support while meeting QoS requirements. The authors in [10], employing the normalized the effective capacity as the performance metric, explored the energy efficiency in the low-power and wideband regimes under QoS constraints. They considered variable-rate/variable-power and variable-rate/fixed-power transmission schemes assuming the availability of channel side information at both transmitter and receiver or only at the receiver. In their study [11], Liu *et al.* focused on the effective capacity and analyzed the resource requirements for Markov wireless channel models while considering fixed-rate transmission schemes and the continuous-time Gilbert-Elliot channel with ON and OFF states. Tang and Zhang in [12] derived the optimal power and rate adaptation techniques that maximize the system throughput under QoS constraints, assuming that the instantaneous channel gains

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are known by both the transmitter and the receiver. In [13], we studied the effective capacity of cognitive radio channels in which the cognitive radio detects the activity of primary users and then performs the data transmission. We assumed that only the receiver has channel side information (CSI), and the transmitter, having no information about the fading coefficients, sends the data at two different fixed rates with two different power levels depending on the results of channel sensing.

In this study, we again focus on the effective capacity of cognitive radio channels but we now study the scenario in which both the receiver and the transmitter have perfect CSI and hence know the instantaneous values of the fading coefficients. We investigate the performance when power and rate adaptation is employed in the system.

The organization of the rest of the paper is as follows. In Section II, we describe the system and the cognitive channel model. In Section III, we discuss channel sensing and provide expressions for the detection and false alarm probabilities. In Section IV, we describe the state transition model for the cognitive radio channel and obtain the effective capacity expression. In Section V, we identify the impact of channel sensing parameters, detection and false alarm probabilities and QoS constraints on the effective capacity through numerical analysis. Finally, in Section VI, we provide conclusions.

## II. System and Cognitive Channel Model

We consider a cognitive radio channel model in which a secondary transmitter aims to send information to a secondary receiver with the possibility of collision with the primary users. Initially, secondary users perform channel sensing, and then depending on the primary users' activity, the transmitter selects its transmission power and rate, i.e., when the channel is busy, the symbol power is $P_1(i)$ and the rate is $r_1(i)$, and when the channel is idle, the symbol power is $P_2(i)$ and the rate is $r_2(i)$, where $i$ denotes the time index. We assume that the data generated by the source is initially stored in the data buffer before being transmitted in frames of duration $T$ seconds over the cognitive wireless channel. The discrete-time channel input-output relation in the $i^{\text{th}}$ symbol duration is given by

$$y(i) = h(i)x(i) + n(i) \quad i = 1, 2, \dots \tag{1}$$

if the primary users are absent. On the other hand, if primary users are present in the channel, we have

$$y(i) = h(i)x(i) + s_p(i) + n(i) \quad i = 1, 2, \dots \tag{2}$$

Above, $x(i)$ and $y(i)$ denote the complex-valued channel input and output, respectively. We assume that the bandwidth available in the system is *B* and the channel input is subject to the following average energy constraints:

$$\mathbb{E}\{|x(i)|^2\} = \mathbb{E}\{P_1(i)\}/B \le \overline{P}_1/B$$
$$\mathbb{E}\{|x(i)|^2\} = \mathbb{E}\{P_2(i)\}/B \le \overline{P}_2/B$$

for all *i*, when the channel is busy and idle, respectively. Since the bandwidth is *B*, symbol rate is assumed to be *B* complex symbols per second, indicating that the average power of the system is constrained by $\overline{P}_1$ or $\overline{P}_2$. In (1) and (2), $h(i)$ is the independent, zero mean, circular, complex Gaussian channel fading coefficient between the cognitive transmitter and the receiver, and has a variance of $\sigma_h^2$. We denote the magnitude of the fading coefficients by $z(i) = |h(i)|^2$. We consider a block-fading channel model and assume that the fading coefficients stay constant for a block of duration $T$ seconds and change independently from one block to another independently.

In (2), $s_p(i)$ represents the sum of the active primary users' faded signals arriving at the secondary receiver. In the input-output relations (1) and (2), $n(i)$ models the additive thermal noise at the receiver, and is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n(i)|^2\} = \sigma_n^2$ for all $i$.

## III. Channel Sensing

If the transmission strategies of the primary users are not known, energy-based detection methods are well-suited for the detection of the activities of primary users. We assume that the first $N$ seconds of the frame duration $T$ is allocated to sense the channel. The channel sensing can be formulated as a hypothesis testing problem between the noise $n(i)$ and the signal $s_p(i)$ in noise. Noting that there are $NB$ complex symbols in a duration of $N$ seconds, this can mathematically be expressed as follows:

$$\begin{aligned} \mathcal{H}_0 \quad &: \quad y(i) = n(i), \quad i = 1, \dots, NB \\ \mathcal{H}_1 \quad &: \quad y(i) = s_p(i) + n(i), \quad i = 1, \dots, NB. \end{aligned} \tag{3}$$

Considering the above detection problem, the optimal Neyman-Pearson detector is given by [2] as follows:

$$Y = \frac{1}{NB} \sum_{i=1}^{NB} |y(i)|^2 \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma. \tag{4}$$

where $\lambda$ is the detection threshold. We assume that $s_p(i)$ has a circularly symmetric complex Gaussian distribution with zero-mean and variance $\sigma_{s_p}^2$. Note that this is an accurate assumption if the signals are being received in a rich multipath environment or the number of active primary users is large. Moreover, if, for instance the primary users are employing phase or frequency modulation, $s_p(i)$ in the presence of even a single primary user in flat Rayleigh fading will be Gaussian distributed[2]. Assuming further that $\{s_p(i)\}$ are i.i.d., we can immediately conclude that the test statistic $Y$ is chi-square distributed with $2NB$ degrees of freedom. In this case, the probabilities of false alarm and detection can be established as follows:

$$P_f = Pr(Y > \gamma | \mathcal{H}_0) = 1 - P\left(\frac{NB\gamma}{\sigma_n^2}, NB\right) \tag{5}$$

$$P_d = Pr(Y > \gamma | \mathcal{H}_1) = 1 - P\left(\frac{NB\gamma}{\sigma_n^2 + \sigma_{s_p}^2}, NB\right) \tag{6}$$

where $P(x, a)$ denotes the regularized lower gamma function and is defined as $P(x, a) = \frac{\gamma(x,a)}{\Gamma(a)}$ where $\gamma(x, a)$ is the lower incomplete gamma function and $\Gamma(a)$ is the Gamma function.

[2]Note that zero-mean, circular, complex Gaussian distributions are invariant under rotation. For instance, if the fading coefficient $h$ is zero-mean, circularly symmetric, complex Gaussian distributed, then so is $he^{j\phi}$ for any random $\phi$.

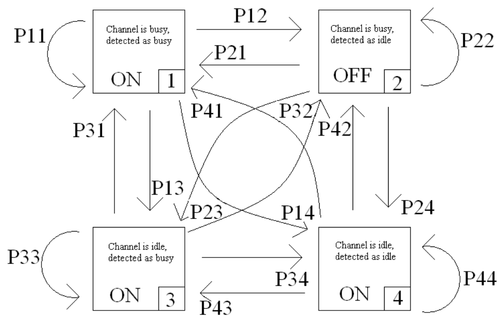


Fig. 1. State transition model for the cognitive radio channel. The numbered label for each state is given on the lower-right corner of the box representing the state.

In the above hypothesis testing problem, another approach is to consider $Y$ as Gaussian distributed, which is accurate if $NB$ is large [3]. In this case, the detection and false alarm probabilities can be expressed in terms of Gaussian $Q$-functions.

## IV. State Transition Model and Effective Capacity

### A. State Transition Model

In this paper, we assume that both the receiver and the transmitter have perfect channel side information and hence perfectly know the instantaneous values of $\{h[i]\}$. In Figure 1, we depict the state transition model for cognitive radio transmission.

First, note that we have the following four possible scenarios:

1) Channel is busy, detected as busy
2) Channel is busy, detected as idle
3) Channel is idle, detected as busy
4) Channel is idle, detected as idle.

In each scenario, we have one state either ON or OFF, depending on whether or not the determined transmission rate exceeds the instantaneous channel capacity. Note that if the channel is detected as busy, the secondary transmitter sends the information with the power policy, $P_1(i)$ (If $P_1(i) = 0$, the secondary transmitter stops the transmission). Otherwise, it transmits with a different power policy, $P_2(i)$.

Since both the transmitter and the receiver are aware of the channel conditions, information is transmitted at rates $r_1(i)$ and $r_2(i)$ which are equal to the estimated optimal channel capacities, depending on the channel being detected as busy or idle, respectively. Note that since the channel can not be sensed correctly all the time, the transmission rates may be below or over the actual channel capacities. Here, we can define the instantaneous transmission powers as $P_1(i) = \mu_1(\theta, z(i))\overline{P}_1$ and $P_2(i) = \mu_2(\theta, z(i))\overline{P}_2$, where $\mu_1(\theta, z(i))$ and $\mu_2(\theta, z(i))$ are the optimal normalized power-adaptation policies for the cases when the channel is busy and idle, respectively, and $\theta$ is *QoS* exponent which is explained in the following section.

Considering the interference $s_p$ caused by the primary users as additional Gaussian noise, we can express the instantaneous channel capacities in the above four scenarios as follows:

$$\begin{aligned}
C_1(i) &= B\log_2(1+\text{SNR}_1(i)z(i)) \text{ (channel busy, detected busy)}\\
C_2(i) &= B\log_2(1+\text{SNR}_2(i)z(i)) \text{ (channel busy, detected idle)}\\
C_3(i) &= B\log_2(1+\text{SNR}_3(i)z(i)) \text{ (channel idle, detected busy)}\\
C_4(i) &= B\log_2(1+\text{SNR}_4(i)z(i)) \text{ (channel idle, detected idle).}
\end{aligned} \tag{7}$$

where

$$\text{SNR}_1(i) = \frac{P_1(i)}{B\left(\sigma_n^2 + \sigma_{s_p}^2\right)} \quad and \quad \text{SNR}_2(i) = \frac{P_2(i)}{B\left(\sigma_n^2 + \sigma_{s_p}^2\right)}$$

$$\text{SNR}_3(i) = \frac{P_1(i)}{B\sigma_n^2} \quad and \quad \text{SNR}_4(i) = \frac{P_2(i)}{B\sigma_n^2}.$$

The optimal transmission rates can be expressed as follows:

$$\begin{aligned}
r_1(i) &= B\log_2(1+\mu_1(\theta, z(i))z(i)\text{SNR}_1)\\
r_2(i) &= B\log_2(1+\mu_2(\theta, z(i))z(i)\text{SNR}_4).
\end{aligned} \tag{8}$$

Using (7) and (8), we observe that we have the ON states when $r_1(i) = C_1(i)$, $r_1(i) < C_3(i)$ and $r_2(i) = C_4(i)$, and the OFF state when $r_2(i) > C_2(i)$. In the OFF state, reliable communication is not achieved, and hence, the information has to be resent. It is assumed that a simple automatic repeat request (ARQ) mechanism is incorporated in the communication protocol to acknowledge the reception of data and to ensure that erroneous data is retransmitted. As depicted in Fig. 1, there are three ON states and one OFF state.

Due to the block fading assumption, state transitions occur every $T$ seconds. When the channel is busy and detected as busy, the probability of staying in the ON state, which is state 1 in Fig. 1, is expressed as $p_{11} = \rho P_d$, where $\rho$ is the prior probability of channel being busy, and $P_d$ is the probability of detection as defined in (6). Since the state transition probabilities are independent of the original states, the other transition probabilities become

$$\begin{aligned}
p_{11} &= p_{21} = p_{31} = p_{41} = \rho P_d\\
p_{12} &= p_{22} = p_{32} = p_{42} = \rho(1-P_d)\\
p_{13} &= p_{23} = p_{33} = p_{43} = (1-\rho)P_f\\
p_{14} &= p_{24} = p_{34} = p_{44} = (1-\rho)(1-P_f).
\end{aligned} \tag{9}$$

Then, we can easily see again due to the block-fading assumption that the $4 \times 4$ state transition probability matrix can be expressed as

$$R = \begin{bmatrix} p_{1,1} & . & . & p_{4,1}\\ . & & & .\\ . & & & .\\ p_{1,4} & . & . & p_{4,4} \end{bmatrix} = \begin{bmatrix} p_1 & . & . & p_1\\ . & & & .\\ . & & & .\\ p_4 & . & . & p_4 \end{bmatrix}.$$

Note that $R$ has a rank of 1. Note also that $r_1(i)(T-N)$ and $r_2(i)(T-N)$ bits are transmitted and received in the states 1 and 3, and 4, respectively, while the transmitted number of

bits is assumed to be zero in state 2.

Moreover, recall that the mean transmit power is upper-bounded by $\overline{P}_1$ and $\overline{P}_2$ when the channel is busy and idle, respectively. Therefore, the optimal power policy needs to satisfy the average power constraints:

$$\int_0^{\infty} \mu_1(\theta, z))f(z)dz \leq 1 \quad \text{and} \quad \int_0^{\infty} \mu_2(\theta, z))f(z)dz \leq 1 \tag{10}$$

where $f(z)$ is the probability density function (pdf) of the power of the channel fading coefficients.

### B. Effective Capacity

Wu and Negi in [4] defined the effective capacity as the maximum constant arrival rate that can be supported by a given channel service process while also satisfying a statistical QoS requirement specified by the QoS exponent $\theta$. If we define $Q$ as the stationary queue length, then $\theta$ is defined as the decay rate of the tail of the distribution of the queue length $Q$:

$$\lim_{q\to\infty} \frac{\log P(Q \geq q)}{q} = -\theta. \tag{11}$$

Hence, we have the following approximation for the buffer violation probability for large $q_{max}$: $P(Q \geq q_{max}) \approx e^{-\theta q_{max}}$. Therefore, larger $\theta$ corresponds to more strict QoS constraints, while the smaller $\theta$ implies looser constraints. In certain settings, constraints on the queue length can be linked to limitations on the delay and hence delay-QoS constraints. It is shown in [5] that $P\{D \geq d_{max}\} \leq c\sqrt{P\{Q \geq q_{max}\}}$ for constant arrival rates, where $D$ denotes the steady-state delay experienced in the buffer. In the above formulation, $c$ is a positive constant, $q_{max} = ad_{max}$ and $a$ is the source arrival rate.

The effective capacity for a given QoS exponent $\theta$ is given by

$$-\lim_{t\to\infty} \frac{1}{\theta t} \log_e \mathbb{E}\{e^{-\theta S(t)}\} = -\frac{\Lambda(-\theta)}{\theta} \tag{12}$$

where $S(t) = \sum_{k=1}^{t} r(k)$ is the time-accumulated service process, and $\{r(k), k = 1, 2, \dots\}$ is defined as the discrete-time, stationary and ergodic stochastic service process. Note that the service rate is $r(k) = r_1(k)(T - N)$ if the cognitive system is in state 1 or 3 at time $k$. Similarly, the service rate is $r(k) = r_2(k)(T - N)$ in state 4. In the OFF state, the determined transmission rate exceeds the instantaneous channel capacity and reliable communication is not possible. Therefore, the service rate in this state is effectively zero.

In the next result, we provide the effective capacity for the cognitive radio channel and state transition model described in the previous section.

*Theorem 1:* For the cognitive radio channel with the state transition model given in Section IV-A, the normalized effective capacity in bits/s/Hz is given by

$$R_E(\text{SNR}, \theta) = \max_{\substack{\mu_1(\theta,z):\mathbb{E}_z\{\mu_1(\theta,z)\}\leq 1 \\ \mu_2(\theta,z):\mathbb{E}_z\{\mu_2(\theta,z)\}\leq 1}}$$
$$-\frac{1}{\theta TB} \log_e \Big[ (\rho P_d + (1-\rho)P_f)\, \mathbb{E}_z\{e^{-(T-N)\theta r_1}\}$$
$$+ (1-\rho)(1-P_f)\mathbb{E}_z\{e^{-(T-N)\theta r_2}\} + \rho(1-P_d)\Big]. \tag{13}$$

*Proof:* In [1, Chap. 7, Example 7.2.7], it is shown for Markov modulated processes that

$$\frac{\Lambda(\theta)}{\theta} = \frac{1}{\theta} \log_e sp(\phi(\theta)R) \tag{14}$$

where $sp(\phi(\theta)R)$ is the spectral radius (i.e., the maximum of the absolute values of the eigenvalues) of the matrix $\phi(\theta)R$, $R$ is the transition matrix of the underlying Markov process, and $\phi(\theta) = \text{diag}(\phi_1(\theta), \dots, \phi_M(\theta))$ is a diagonal matrix whose components are the moment generating functions of the processes in given states. The rates supported by the cognitive radio channel with the state transition model described in the previous section can be seen as a Markov modulated process and hence the setup considered in [1] can be immediately applied to our setting. Since the transmission rates are non-random and fixed in each state in the cognitive channel, we can easily find that

$$\phi(\theta) = \text{diag}\Big\{\mathbb{E}_z\{e^{-(T-N)\theta r_1}\}, 1, \mathbb{E}_z\{e^{-(T-N)\theta r_1}\}, \mathbb{E}_z\{e^{-(T-N)\theta r_2}\}\Big\}. \tag{15}$$

Then, we have

$$\phi(\theta)R = \begin{bmatrix} \phi_1(\theta)p_1 & . & . & \phi_1(\theta)p_1 \\ . & & & . \\ . & & & . \\ \phi_4(\theta)p_4 & . & . & \phi_4(\theta)p_4 \end{bmatrix}.$$

Since $\phi(\theta)R$ is a matrix with unit rank, we can readily find that

$$sp(\phi(\theta)R) = \phi_1(\theta)p_1 + ... + \phi_4(\theta)p_4 \tag{16}$$
$$= (p_1 + p_3)\mathbb{E}_z\{e^{-(T-N)\theta r_1}\} + p_2 + p_4\mathbb{E}_z\{e^{-(T-N)\theta r_2}\}. \tag{17}$$

Then, combining (17) with (14) and (12), and noting that choice of the power policies $\mu_1(\theta, z)$ and $\mu_2(\theta, z)$ can be optimized leads to the effective capacity formula given in (13). □

Having obtained the expression for the effective capacity, we now derive the optimal power adaptation strategies that maximize the effective capacity.

*Theorem 2:* The optimal power adaptation policies that maximize the effective capacity are given by

$$\mu_1(\theta, z) = \begin{cases} \frac{1}{\text{SNR}_1}\left(\frac{1}{\gamma_1^{\frac{1}{a+1}}} \frac{1}{z^{\frac{a}{a+1}}} - \frac{1}{z}\right), & z > \gamma_1 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

and

$$\mu_2(\theta,z)=\begin{cases}\frac{1}{\text{SNR}_4}\left(\frac{1}{\gamma_2^{\frac{1}{a+1}}}\frac{1}{z^{\frac{a}{a+1}}}-\frac{1}{z}\right), & z>\gamma_2\\ 0, & \text{otherwise.}\end{cases} \tag{19}$$

where $a=(T-N)B\theta/\log_e 2$. $\gamma_1$ and $\gamma_2$ are the threshold values in the power adaptation policies and they can be found from the average power constraints in (10) through numerical techniques.

*Proof:* Since logarithm is a monotonic function, the optimal power adaptation policies can also be obtained from the following minimization problem

$$\min_{\substack{\mu_1(\theta,z):\mathbb{E}_z\{\mu_1(\theta,z)\}\leq 1\\ \mu_2(\theta,z):\mathbb{E}_z\{\mu_2(\theta,z)\}\leq 1}}\Big\{(\rho P_d+(1-\rho)P_f)\,\mathbb{E}_z\{e^{-(T-N)\theta r_1}\} + (1-\rho)(1-P_f)\mathbb{E}_z\{e^{-(T-N)\theta r_2}\}\Big\}. \tag{20}$$

It is clear that the objective function in (20) is strictly convex and the constraint functions in (10) are linear with respect to $\mu_1(\theta,z)$ and $\mu_2(\theta,z)$. Then, forming the Lagrangian function and setting the derivatives of the Lagrangian with respect to $\mu_1(\theta,z)$ and $\mu_2(\theta,z)$ equal to zero, we obtain

$$\left\{\lambda_1-\frac{a\text{SNR}_1 z\,[\rho P_d+(1-\rho)P_f]}{[1+\mu_1(\theta,z)z\text{SNR}_1]^{a+1}}\right\}f(z)=0 \tag{21}$$

$$\left\{\lambda_2-\frac{a\text{SNR}_4 z\,(1-\rho)\,(1-P_f)}{[1+\mu_2(\theta,z)z\text{SNR}_4]^{a+1}}\right\}f(z)=0 \tag{22}$$

where $\lambda_1$ and $\lambda_2$ are the Lagrange multipliers. Defining $\gamma_1=\frac{\lambda_1}{[\rho P_d+(1-\rho)P_f]a\text{SNR}_1}$ and $\gamma_2=\frac{\lambda_2}{(1-\rho)(1-P_f)a\text{SNR}_4}$, and solving (21) and (22), we obtain optimal power policies given in (18) and (19). □

The optimal power allocation schemes identified in Theorem 2 are similar to that given in [6]. However, in the cognitive radio channel, we have two allocation schemes depending on the presence or absence of active primary users. Note that the optimal power allocation in the presence of active users, $\mu_1(\theta,z(i))=\frac{P_1(\theta,z(i))}{\overline{P}_1}$, has to be performed under a more strict average power constraint since $\overline{P}_1<\overline{P}_2$. Note also that under certain fading conditions, we might have $\mu_1(\theta,z(i))>\overline{P}_1$, causing more interference to the primary users. Therefore, it is also of interest to apply only rate adaptation and use fixed-power transmission in which case we have $\mu_1(\theta,z(i))=\mu_2(\theta,z(i))=1$. We can immediately see from the result of Theorem 1 that the effective capacity of fixed-power/variable-rate transmission is

$$R_E(\text{SNR},\theta)=-\frac{1}{\theta TB}\log_e\Big[(\rho P_d+(1-\rho)P_f)\,\mathbb{E}_z\{e^{-(T-N)\theta r_1}\} +(1-\rho)(1-P_f)\mathbb{E}_z\{e^{-(T-N)\theta r_2}\}+\rho(1-P_d)\Big] \tag{23}$$

where $r_1=B\log_2(1+z\text{SNR}_1)$ and $r_2=B\log_2(1+z\text{SNR}_4)$.

## V. Numerical Results

In this section, we present the numerical results. In Figure 2, we plot the effective capacity with respect to energy detection threshold $\lambda$ for different values of detection duration $N$. We compare the probabilities of false alarm, $P_f$ and detection, $P_d$. The channel bandwidth is assumed to be 10kHz and the average input $SNR$ values, when the channel is correctly detected, are $SNR_1=0$dB and $SNR_4=10$dB. The QoS exponent $\theta=0.01$. The channel is assumed to be busy with an average probability of $\rho=0.1$ and the duration of the block is $T=0.1$s. As we see in Fig. 2, $P_d$ and $P_f$ depend on the duration of channel detection. With increasing channel detection duration, we get more concrete $P_d$ and $P_f$ values. We can easily see that with a reasonable channel detection duration, there is an optimal energy detection threshold interval, which can be observed to be in between the values, 1.2 and 1.7. In addition, we should note that there is also an optimal channel detection duration. As we can see in Fig. 2, when $N=0.01$s, we have higher effective capacity than we have when $N=0.02$s. It is because the duration allocated for data transmission is decreasing. The maximum effective capacity is $0.11$ bits/s/Hz, which is obtained when $N=0.01$s and $\lambda$ is in between 1.2 and 1.7. With increasing energy detection threshold, the effective capacity decreases sharply, which is because the time allocated for the OFF state in which there is no reliable data transmission increases.

In Fig. 3, the input SNR values are kept the same as the ones used in Fig. 2 while we change the QoS exponent to $\theta=1$ and the bandwidth to $B=1$kHz. Again, the block duration is $T=0.1$s. We consider 4 different detection durations and compare the effective capacity results. As we can easily realize, $P_d$ and $P_f$ show similar patterns as in Fig. 2. But, since the bandwidth is small compared to the one in Fig. 2, the quality of channel detection has decreased. Therefore, the highest effective capacity rate is obtained not when $P_d=1$ and $P_f=0$. Hence, there is a tradeoff between the effective capacity rate and disturbance caused to the primary users.

In Fig. 4, we plot the effective capacity with respect to decay rate, $\theta$, where we consider the same input SNR values as we have in Fig. 2 and Fig. 3. The channel bandwidth is $B=1$kHz, block duration is $T=0.1$s, and the energy detection threshold is $\lambda=1.4$. We obtained the effective capacity both with and without optimizing the transmission power adaptation policy. For a given channel detection duration, the effective capacity is decreasing as $\theta$ increases, as expected. On the other hand, for fixed $\theta$ and $\lambda$ values, we obtain different optimum channel detection duration values. For instance, for lower decay rates $\theta$, we obtain higher effective capacity when $N=0.02$s. For higher decay rate values, choosing $N=0.05$s provides higher effective capacity values. We should also note that since $P_d$ and $P_f$ are independent of the decay rate, for each detection duration there exist particular $P_d$ and $P_f$ values and they are constant for all decay rates. Finally, we can notice that for high values of $\theta$, power adaptation does not provide much gain in terms of the effective capacity.

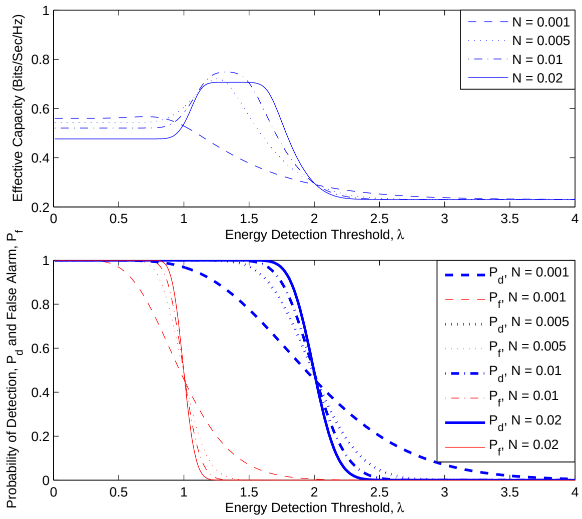


Fig. 2. Effective Capacity and $P_f$-$P_d$ vs. Channel Detection Threshold, $\lambda$.

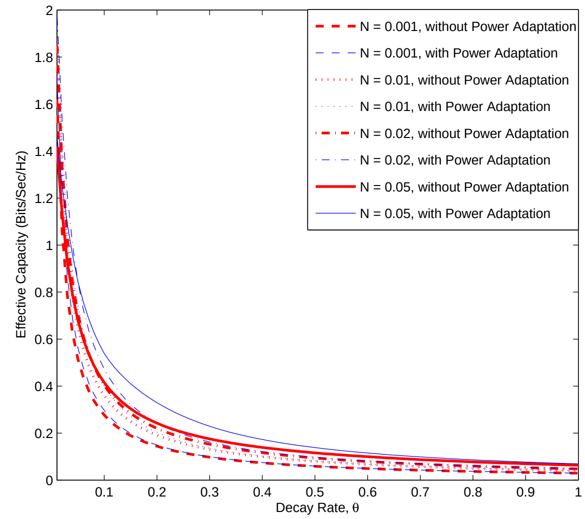


Fig. 4. Effective Capacity vs. Decay Rate, $\theta$.

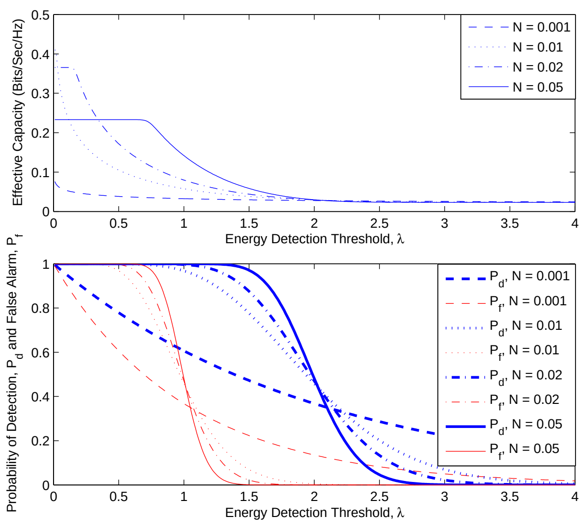


Fig. 3. Effective Capacity and $P_f$-$P_d$ vs. Channel Detection Threshold, $\lambda$.

## VI. Conclusion

In this paper, we have analyzed the effective capacity of cognitive radio channels in order to identify the performance levels and to determine the interactions between throughput and channel sensing parameters in the presence of QoS constraints. We have assumed that both the receiver and the transmitter have CSI. We have initially constructed a 4-state-transition model for cognitive transmission and then obtained expressions for the effective capacity. This analysis is conducted for variable transmission rates and variable transmission powers. Through numerical results, we have investigated the impact of channel sensing duration and threshold, detection and false alarm probabilities, and QoS limitations on the throughput. Several insightful observations are made. For instance, we have seen that the channel sensing duration and threshold have great impact on the effective capacity. Also, we have noted that the gains provided by power adaptation diminishes as $\theta$ increases.

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